

Concrete Representational Abstract

For: Students in Grade K and above who have not reached the benchmark/ target scores on assessments or who have difficulty with word problems/ math application problems. Good candidates for this intervention are students who do not seem to have a good understanding of math concepts when presented initially at an abstract level.

Materials:

- Manipulative or concrete objects: Materials will vary depending on the math strand of math concepts being addressed. Examples are below:

Math Strand or Concept Being Addressed:	Examples of Appropriate Manipulatives:
Early Numeracy (counting, one-to-one correspondence, quantity)	Counters (blocks, chips, children, toys, etc.), dry beans, small candies, popsicle sticks, etc.
Measurement	Ruler, yardstick, scale balance, trundle wheel, thermometer, cups, geoboards (for area, perimeter), etc.
Base 10 System/ Place Value	Unifix cubes, beansticks, base-ten blocks, popsicle stick bundles, abacus, poker chips or beads (where color indicates value), place value mats, etc.
Multiplication/ Division	Counters, trays, egg cartons, cups, other objects used for dividing/ separating (paper plates, mats, etc.)
Positive and Negative Integers	Counters (one set light colored for positive numbers, one set dark colored for negative) Note: When adding positive and negative integers, the student matches pairs of dark and light colored objects. The color and number of objects remaining represent the solution.
Fractions	Fraction pieces (circles, half-circles, etc.), strips (wholes, halves, thirds, etc.), or blocks or stacks ('1/2" block is twice the height of "1/4" block, etc.)
Geometry	Geoboards, rubber bands/ string, concrete objects representing 2-3 dimensional shapes
Beginning Algebra	Containers (representing the variable of "unknown") and counting objects (representing integers) e.g. paper dessert plates & beans, small clear plastic beverage cups, counting chips, candy pieces, etc.

- Plain paper for drawing
- Student work from the curriculum program, or other work the student is using for practice

Recommended Duration and Frequency: This intervention should be conducted at least 3 times per week for 20-30 minutes per session. Monitor the student's progress once a week. When the student's score is at the benchmark goal, the intervention may be discontinued.

Steps for Intervention:

Note: The purpose of teaching through a concrete-to-representational-to-abstract sequence of instruction is to ensure students truly have a thorough understanding of the math concepts/ skills they are learning. When students who have math learning problems are allowed to first develop a concrete understanding of the math concept/skill, then they are much more likely to perform that math skill and truly understand math concepts at the abstract level.

CONCRETE LEVEL

1. Model each math concept/ skill/ problem using "explicit teacher modeling" (see below) with concrete materials (see examples above, depending on the strand in which the student is working). Model/demonstrate for the student using similar types of problems at least 3 times.
2. When the student is ready, provide him/her with at least 3 opportunities for guided practice (with peer or teacher scaffolding/ assistance) using concrete materials. Complete the Recording Sheet (attached), circling "C" for "Concrete" and circling "Guided Practice" in the "Level of Support" column, making notes indicating the amount of assistance needed.
3. When the student is ready, provide him/her with at least 3 opportunities for independent practice (with no assistance) using concrete materials. Complete the Recording Sheet (attached), circling "C" for "Concrete" and circling "Independent Practice" in the "Level of Support" column, making notes indicating the student's level of success without help. When the student is at least 90% accurate with concrete objects 3 days in a row, move to the Representational Level.

REPRESENTATIONAL LEVEL

1. Model each math concept/ skill/ problem using "explicit teacher modeling" (see below) at the representational (semi-concrete) level, which involves drawing pictures that represent the concrete objects previously used (e.g. tallies, dots, circles, stamps that imprint pictures for counting). Model/ demonstrate for the student using similar types of problems at least 3 times.
2. When the student is ready, provide him/her with at least 3 opportunities for guided practice (with peer or teacher scaffolding/ assistance) using drawings. Complete the Recording Sheet (attached), circling "R" for "Representational" and circling "Guided Practice" in the "Level of Support" column, making notes indicating the amount of assistance needed.

3. When the student is ready, provide him/her with at least 3 opportunities for independent practice (with no assistance) using drawings. Complete the Recording Sheet (attached), circling "R" for "Representational" and circling "Independent Practice" in the "Level of Support" column, making notes indicating the student's level of success without help. When the student is at least 90% accurate with drawings 3 days in a row, move to the Abstract Level.

ABSTRACT LEVEL

1. Model each math concept/ skill/ problem using "explicit teacher modeling" (see below) at the abstract level, which involves using numbers or math symbols only. Model/ demonstrate for the student using similar types of problems at least 3 times.
2. When the student is ready, provide him/her with at least 3 opportunities for guided practice (with peer or teacher scaffolding/ assistance) at the abstract level. Complete the Recording Sheet (attached), circling "A" for "Abstract" and circling "Guided Practice" in the "Level of Support" column, making notes indicating the amount of assistance needed.
3. When the student is ready, provide him/her with at least 3 opportunities for independent practice (with no assistance) at the Abstract Level. Complete the Recording Sheet (attached), circling "A" for "Abstract" and circling "Independent Practice" in the "Level of Support" column, making notes indicating the student's level of success without help. When the student is at least 90% accurate at the Abstract Level 3 days in a row and is at benchmark/ goal, discontinue the intervention or begin again at the Concrete Level with a different strand or kind of problem.

Explicit Teacher Modeling

1. Ensure that your students have the prerequisite skills to perform the skill.
2. Break down the skill into logical and learnable parts (Ask yourself, "What do I do and what do I want to think as I perform the skill?").
3. Provide a meaningful context for the skill (e.g. word or story problem suited to the age and interests of your students).
4. Provide visual, auditory, kinesthetic (movement), and tactile (manipulative) means for illustrating important aspects of the concept/ skill (e.g. visually displayed word problem and equation, orally cue students by varying vocal intonations, point, circle, highlight computation signs or important information in story problems).
5. "Think aloud" as you perform each step of the skill (i.e. say aloud what you are thinking as you problem-solve).
6. Link each step of the problem-solving process (e.g. restate what you did in the previous step, what you are going to do in the next step, and why the next step is important to the previous step).
7. Periodically check student understanding with questions, remodeling steps when there is confusion.
8. Maintain a lively pace while being conscious of student information processing difficulties (e.g. need additional time to process questions).
9. Model a concept/ skill at least three times before moving on to "Guided Support." Be sure the student is ready to try the skill before you remove the modeling.

Concrete Representational Abstract (CRA) Integrity Check

Interventionist: _____ Date: _____ Grade Level: _____ Tier: _____

Integrity Monitor: _____

Descriptor - Student	Yes	No	N/A
Student has scored below benchmark on the universal screening or has difficulty with computation or applied problems as demonstrated on classroom tests or activities.			
Student is in Grade K or higher.			

Descriptor – Materials	Yes	No	N/A
Student has a sheet of problems without answers, appropriate concrete manipulatives for the task (if at the Concrete Level), drawing paper (if at the Representational Level), or no other materials (if at the Abstract Level).			
Interventionist has a recording sheet.			

Descriptor – Interventionist	Yes	No	N/A
The Interventionist maintains an environment conducive to task completion (quiet, manages behavior issues, engages student, etc.)			
The Interventionist is implementing the intervention at the appropriate level (C, R, or A) depending on the student's needs.			
The Interventionist models the task (if at the first step of either the C, R, or A Level) using the "Explicit Teacher Modeling" steps and the appropriate materials, if any, in at least 3 opportunities.			
The Interventionist provides guided practice for the task (teacher or peer assistance, if at the second step of either the C, R, or A Level) using appropriate materials, if any, in at least 3 opportunities.			
The Interventionist provides independent practice for the task (no assistance, if at the third step of either the C, R, or A Level) using no materials other than the set of problems, in at least 3 opportunities.			
The Interventionist dates and makes notes on the Recording Sheet regarding student performance and any difficulty the student had.			
Student's progress is monitored using FastBridge weekly (if on tier 3) or every other week (if on tier 2).			
The Interventionist either discontinues the intervention or begins again at the Concrete Level with a different strand or kind of problem when the student is at least 90% accurate at the Abstract Level 3 days in a row and is at benchmark goal.			

CRA Intervention Integrity Check Summary: _____ of _____ applicable components are observed.

Notes: